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ABSTRACT

of the dissertation for the degree of Doctor of Philosophy

MANAGEMENT PROBLEMS IN QUEUING SYSTEMS WITH CYCLIC SERVICE MOMENTS

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Field of science:	Mathematics
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Rationale and development. In the early twentieth century, the solution to many practical problems for complex systems led to the emergence of a new research field known as queue theory and queuing system. The methods and approaches of mathematics, especially the theory of probability, have been successfully used in this regard, thus giving an enormous impetus to the development of this field. Modern queue theory is considered to be an advanced branch of the theory of probability.

The main objects of queuing systems are (included in the system) customers and devices (servers) (serving customers). Such systems have various parameters, such as the average length of the queue, the average waiting time of customers before service, the average cost of service, the distribution of employment time, etc. The mentioned parameters play an important role in various practical application areas such as industry, transport, communication systems, etc.

The primary purpose of the listed studies is to assess the various features of the systems and to create different control strategies for optimization (minimization or maximization) of the selected useful indicators - the length of the queue, waiting time, duration of employment, the number of customers served, etc. Thus, the calculation of various parameters for queuing system and management problems plays an essential role in their research and is currently attracting many scientists' attention. Until the end of the twentieth century, research in this field was mainly analytical, so the study of complex systems faced particular difficulties. However, the widespread use of computers in the second half of the twentieth century relatively eliminated these difficulties and led to the creation of new directions. Computer capabilities and methods are now successfully used in various fields of science, and even new scientific directions related to computers have emerged (computer modeling, computer statistics, (MCMC- Markov Chain and Monte Carlo).

In recent years, the world economy has been declining for various reasons. The development of communication, transport, and communication systems plays a vital role in the growth of the world economy. Queuing system with moving devices and their mathematical models closely reflect the operation of these systems. In this regard, the study of queuing systems with moving devices and their mathematical models is one of the most relevant and essential issues.

In the early years, issues of the queuing system were related to customer service (telephone calls) in telephone systems and then to systems with complex structures. The first articles on queuing system belong to Agner Erlang, an employee of the Copenhagen telephone company in the early twentieth century. Later, Agner Erlang became a well-known scientist. Significant contributions to the queue theory were made by professors of the Moscow State University named after Lomonosov A.Ya. Khinchin[1] and B.V. Gnedenko[2], who created a world-famous scientific school. Then the researches of many scientists (E.Cinlar, N.Prabhu, L.Kleinrock, T.Saati, Yu.Belyaev, A.Borovkov, A.Solovyev, V.Zolotarev, V.Korolyuk, I.Kovalenko, etc.) made significant contributions to the development of this field and have solved many vital issues. In modern queue theory, systems with moving devices are of particular interest since these models can be used, for example, in transport, public transport, logistics problems, communication systems, computer networks, and various other applications.

In the development of queue theory in the second half of the twentieth century, the results and approaches of theory of probability, random processes and other fields of mathematics are used in traffic flow management, airports, tunnel traffic management, transport and communication systems, computer networks and other systems.

The study of such systems has led to the emergence of new research areas, as they go beyond the methods and approaches of classical queue theory. Although these systems have a complex

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[1] Хинчин, А.Я. Избранные труды по теории вероятностей / А.Я.Хинчин – Москва: ТВП, – 1995. – 552 с.

[2] Гнеденко, Б.В. Беседы о теории массового обслуживания / Б.В.Гнеденко – Москва: Либроком, – 1973. – 72 с.

structure, they all have one thing in common - they are queuing systems with moving devices. In the literature, such systems are sometimes called "shuttle" systems.

Typical examples of queuing systems with moving devices are the movements of transport units, various goods, cargoes, data and computer networks, communication systems, the service of employees to several pieces of equipment, and so on.

Systems with moving devices have particular parameters. For example, intensity in transport systems (number of vehicles entering the system during a specific period), and traffic flow density (number of vehicles on one line of the road), road capacity, road diagram (dependence of the intensity on density), average waiting time for demands (average waiting time before service), full-service life (total waiting and service life), energy costs in communication systems, etc.

The study of such systems leads to the construction and study of new mathematical models, and new research methods are formed within such mathematical models. Mathematical models with moving particles are widely used in this direction. Such models represent the operation of various moving queuing systems (transport, communication systems, etc.) and allow them to obtain new results.

One of the first mathematical models adequately expressing transport systems was proposed by Yu.K. Belyayev[3] in 1969. In this model, only two particles move. Models with increasing particle numbers are more complex and pose new challenges. U. Zahle[4] constructed mathematical models of a large number of moving particles, and Belyayev's results were generalized for such models. As in the articles mentioned above, the motions of the particles in

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[3] Беляев, Ю.К. Сравнительный анализ простейших систем вертикального транспорта / Ю.К. Беляев, А.Г.Гаджиев, Ю.И.Громак [и др.] // Известия Академии Наук СССР: Техническая кибернетика, Москва: – 1977. №3, – с. 97-103.

[4] Целе, У.Н. Обобщение модели движения без обгона // – Москва: Известия Академии Наук СССР, Техническая кибернетика, – 1972, т.3, №5, – с. 100-103.

the mathematical models proposed in the dissertation depending on the distance between them, but unlike Yu.K. Belyayev and U. Zahle , each particle can move at either V_1 or V_2 speed, $V_1 < V_2$. The movement of the particles at different speeds complicates the models, but at the same time allows them to express practical issues more closely. Each particle's speed depends on the distance to the next particle (in the direction of movement). There can be only one particle at each point in a circle. The construction of the mathematical models proposed below will show that particles cannot cross each other in motion. Such models represent the processes that take place in transport, communication systems, biology (impulse transmission), and other systems.

The first mathematical models of moving particles in a straight-line representing queuing system with moving devices[5] have been proposed. An unexpected effect was found on these models - the random rotation of the particle under consideration. This result allows us to build a schedule called a road diagram in transport and explain the causes of traffic jams.

Since the mathematical models of particles moving on a circle are more complex, the use of the methods proposed in requires additional research for these models. Such models have been studied[6] and necessary results have been obtained for various characteristics of the systems (waiting time of the particle at a certain point on the circle, the density of particles on the circle, intensity, and other parameters).

Modern queuing systems with devices in motion usually have

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[5] Hajiyev, A.H. Minimization of the waiting time in systems with recurrent services // – Moscow: Bulletin of Moscow University, – 1980. vol.37, No.03, – p. 19-23.

[6] Hajiyev, A.H. Delays reducing the waiting time in queuing systems with cyclic services // – Stockholm: Scandinavian Journal of Statistics, – 1985. vol.12, No.04, – p. 301-307.

Ignall, E.R., Kolesar, P.S. Operating characteristics of a simple shuttle under local dispatching rules // – Catonsville: Operation Research, – 1972. vol.20, No.06, – p. 1077-1088.

Osuna, E.E., Newell, G.F. Control stratrgies for an idealized public transportation system // – Catonsville: Transportation Science, – 1972. vol.6, No.01, – p. 52-72.

limited resources, such as the waiting time for traffic and limited access to the road, computer networks - limited memory, etc. Thus, it is natural and necessary to explore different control strategies for such systems, which allows them to operate more effectively. Although such issues have been studied[7], the use of these results for a wide range of systems poses specific difficulties. Therefore, there was a great need to create new universal methods that take into account the nature of the systems. The dissertation pays special attention to such issues. The average waiting time of the demands, the inclusion of the control function and finding the optimal modes, and other issues, which are the main characteristics of the systems, are important both from a theoretical and practical point of view.

For example, an increase in the number of vehicles in transport systems can sometimes lead to traffic jams and substantial losses. In the United States, traffic jams in the Lincoln Tunnel have caused high material losses. Required statistical data[8] proposes to eliminate such problems, albeit partially. In such systems, management issues and selection of optimal modes are essential. The dissertation includes a control function for queuing systems with some devices in motion and shows what measures should be taken to prevent traffic jams.

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[7] Гаджиев, А.Г. О случайном блуждании частиц по кольцу // – Москва: Математические заметки, – 1990. т.47, №6, – с. 140-143.

Гаджиев, А.Г. Модель движение частиц по замкнутому контуру без обгона // – Москва: Известия Академии Наук СССР: Техническая кибернетика, – 1976. № 5, – с. 79-84.

Belyaev, Y.K., Hajiyev, A.H. Mathematical models of lift systems and their Simulation // Proceedings of the Thirteenth International Conference on Management Science and Engineering Management, – Ontario: – 05-08 August, – 2019, – p. 507-519.

Ignall, E.R., Kolesar, P.S. Optimal dispatching of an infinite capacity shuttle: control at a single terminal // – Catonsville: Operation Research, – 1974. vol.22, No.05, – p. 1008-1025.

[8] Пройдакова, Е. В., Федоткин, М. А. Достаточное условие существования стационарного распределения выходных потоков в системе с циклическим управлением // – Нижний Новгород: Вестник Нижегородского Университета: Математика, – 2006. т.42, № 3, – с. 118-126.

In 1969, Yu. Belyayev[9] proposed simplified mathematical models of moving particles. This article shows that although the motion of particles depends on each other, each particle's motion under consideration corresponds to the random binomial rotation. This unexpected result (Belyayev effect) allowed the calculation of several characteristics of the system and the construction of a road diagram. Based on these results, it was possible to make the necessary recommendations to eliminate the traffic. The effect obtained can be called the "Belyayev effect" and allows us to estimate traffic jams and calculate some road features. The Belyayev effect is generalized for more extensive classes. It is noteworthy that this result was used in the formation of the transport system on the Moscow ring road[10].

The motion of particles in closed trajectories (for example, moving in a circle) describes the behavior of public transport (buses, trains, trams) and leads to more complex mathematical problems as the particles interfere with each other. To study them, there is a need to build new mathematical models and develop new approaches. Consider some mathematical models of particles moving in a circle[11]. Necessary and sufficient conditions are provided for these models for the Belyayev effect to occur. It is shown that the distance between the particles can be approximated by Poisson, binomial,

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[9] Беляев, Ю.К. Сравнительный анализ простейших систем вертикального транспорта / Ю.К. Беляев, А.Г.Гаджиев, Ю.И.Громак [и др.] // Известия Академии Наук СССР: Техническая кибернетика, Москва: – 1977. №3, – с. 97-103.

[10] Беляев, Ю.К., Громак, Ю.И., Малышев, В.А. Об инвариантных случайных булевских полях // – Москва: Математические заметки, – 1969. т.6, № 5, –с. 555-566.

[11] Гаджиев, А.Г., Маммадов, Т.Ш. Математические модели движущихся частиц и их приложения // – Москва: Журнал имени А.Н.Колмогорова: Теория вероятностей и ее применения, – 2011. т.56, №4, – с. 1-14.

Замятин, А.А., Малышев, В.А. Накопление на границе для одномерной стохастической системы частиц // – Москва: Проблемы передачи информации, – 2007. т.43, №4, – с. 68-82.

Hajiyev, A.H. Shuttle Systems // – Moscow: Dokladi Mathematics RAS, – 2001. vol.380, No.05, – p. 583-585.

geometric distributions, depending on the parameters of particle motion. The study of two-line systems leads to more complex models, and such issues have been studied[12]. The issue of moving through a single-line tunnel and its planning, which is called the "glass neck" system in the literature, has been studied[13], where the optimal control model is proposed.

An essential problem in queue theory is that it has provided significant gains to the efficiency index and is a management issue easily implemented in practice (for example, delays in starting service). The management function should be selected so that it is easy to implement in practice and that the requirements are met before service and that accidents occur during waiting and service time. Mathematical models representing such systems have a very complex structure. The study of some mathematical models of such systems by analytical methods is given[14]. Surprisingly, delaying the service moments of demands as a management function has been suggested in these studies. It has been given that for some systems, such a management function improves service (the average waiting time of demand reduces), and a form of optimal management function has been proposed. However, such systems in the research in this area should first look at which included management function

[12] Deb, R. K. Optimal dispatching of a finite capacity shuttle // – Catonsville: Management Science, – 1978. vol.24, No.13, – p. 1362-1372.
Hajiyev, A.H., Abdullayeva, N.A., Mammadov, T.S. Mathematical Models of Queues with Moving Servers: Simple Vertical Transportation Systems // Proceedings of the Sixth International Conference on Management Science and Engineering Management (Part of the Lecture Notes in Electrical Engineering book series (LNEE, volume 185)), – Islamabad: – 11-14 Novermber, – 2012, – p. 529-547.

[13] Moeschlin, O.E., Poppinga, C.H. Controlling traffic lights at a bottleneck with renewal arrival processes // – Baku: Proceedings of the Institute of Mathematics and Mechanics, Azerbaijan National Academy of Science, – 2001. vol.XIV, No.14, – p. 187-194.

[14] Беляев, Ю.К. Предельные теоремы для редеющих потоков // – Москва: Теория вероятностей и ее применения, – 1963. т.8, №2, – с. 175-184.
Гаджиев А.Г., Мамедов Т.Ш. Циклические системы с задержкой обслуживания. // Доклады Академии Наук России, Математика, – 2009. т. 426, № 1, – с. 15-19.

can improve the service. Such conditions are given[15], and the necessary and sufficient conditions for the inclusion of the management function has been found. The dissertation considers such issues for more complex systems and finds optimal regimes for some cases.

Another critical issue for models of moving particles is to select the optimal number. If the number of particles increases, the time required for service in the system naturally decreases. However, the increasing number of particles is likely to lead to a decrease in the particles' velocity, and in such cases, the waiting time for the demands may increase. In such cases, different modes of movement appear in the system, which arises the issue of choosing the optimal model from these modes. Some mathematical models of moving particles have been constructed[16], and the modes of motion created in such models have been found.

Building adequate mathematical models that describe real systems is a complex process and requires theoretical research and empirical observations. Thus, mathematical models arise from practice, are formed theoretically, and corrected by empirical observations.

The collection of empirical data plays an essential role in the study of public service systems, which contributes to the construction and study of mathematical models.

For example, empirical data on the Hudson Bay (the USA) show that companies are losing large sums of money when

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[15] Беляев, Ю.К. Предельные теоремы для редеющих потоков // – Москва: Теория вероятностей и ее применения, – 1963. т.8, №2, – с. 175-184.
Беляев, Ю.К., Громак, Ю.И., Малышев, В.А. Об инвариантных случайных булевских полях // – Москва: Математические заметки, – 1969. т.6, № 5, –с. 555-566.
Хинчин, А.Я. О пуассоновских потоках случайных событий // – Москва: Теория вероятностей и ее примения, – 1956. т.1, №3, – с. 320-327.
[16] Deb, R. K., Serfozo, R. F. Optimal control of batch service queues // – Cambridge: Advances in Applied Propability, – 1973. vol.5, No.02, – p. 340-361.
Deb, R. K., Schmidt, C. P. Optimal average cost policies for the two- terminal shuttle // – Catonsville: Management Science, – 1987. vol.33, No.05, – p. 662-669.

delivering cargo and other materials due to disturbing facts such as traffic jams, long queues, etc. Therefore, the construction and research of such mathematical models is significant for practical applications and can be used to implement large transport projects.

There are different types of controls in systems where the devices are in motion, but as mentioned above, it is necessary to create a control policy that provides significant gains in the selected useful indicators and can be easily implemented in practice. For example, bus systems always have schedules, but in reality, buses arrive at the bus stops with some deviations. Therefore, special control functions should be included to eliminate the adverse effects of these deviations and optimize movement. These issues are also included in the dissertation.

First, the introduction of delay functions in the queuing systems was proposed by Yu. Belyayev. Then the systems obtained numerical results by computer modeling given[17], where the delay function reduces the average waiting time of the demands in some systems. Mathematical models of particles moving in a straight line are given in the literature[18], and the necessary and sufficient conditions for the inclusion of the delay function by analytical methods have been offered. Such results are generalized for more

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[17] Kaklauskas, L.A., Sakalauskas, L.B. On network traffic statistical analysis // – Vilnius: Lithuanian Mathematical Journal, – 2008. vol.48, No.01, – p. 314-319.
Kerner, B.S., Klenov, S.L., Hiller, A.H. Criterion for traffic phases in single vehicle data and empirical test of a microscopic three-phase traffic theory // – Harvard: Journal of Physics A: Mathematical and General, – 2006. vol.39, No.09, – p. 2001-2020.

[18] Adan, I.S., Kok, T.G. Waiting time characteristics in cyclic queues // – Michigan: Probability in the Engineering and Informational Science, – 2004. vol.18, No.03, – p. 299-313.
Bacelli, F.A., Foss, S.P. Mathematical methods of stochastic network analysis // – Paris: French-Russian Institute for Applied Mathematics and Computer Science Transactions, – 2001. No.3, – p. 38-44.
Kerner, B.S., Klenov, S.L., A theory of traffic congestion on moving bottlnecks // – Harvard: Journal of Physics A: Mathematical and Theoretical, – 2010. vol.43, No.10, – p. 425-435.

complex systems[19].

If the mathematical model correctly describes practical systems' behavior, research is usually more complex, and the application of analytical approaches faces some difficulties. Other approaches need to be used and developed to investigate such models effectively. For example, Monte Carlo, Markov Chains, and Monte Carlo - [MCMC - Markov Chain and Monte Carlo CMC], intensive computational statistics methods, etc. Modern IT (Information Technology) capabilities, modeling approaches can be used here. The use of these methods to investigate complex queuing systems allows the discovery of some unexpected but exciting and significant effects in these systems from a practical point of view. These results are a sign of the successful continuation of analytical research.

More complicated mathematical models are created by looking at the motion of particles moving on a plane. Mathematical models called Sinai billiard balls[20] are known in the literature. In these models, a particle (a billiard ball) moves between the square-shaped walls, and there is an obstacle in the middle of the squares. After a few strokes, the particle's motion becomes chaotic, and its coordinates are practically impossible to calculate. Even the motion of a particle cannot be calculated by computer modeling. It is because the chaotic motion in these models is multidimensional, and it is complicated to study them since the motion of the particles is in different random directions. Mathematical models of particles moving on a circle are one-dimensional, and it is possible to obtain some analytical results for such models. More sophisticated models

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[19] Афанасьева, Л.Г., Булинская, Е.В. Некоторые задачи для потоков взаимодействующих частиц // – Москва: Сборник, посвященный 70-летию ректора МГУ В.А.Садовничего. Современные проблемы математики и механики, – 2009. т.2, №4, – с. 55-68.
Adan, I.S., Kok, T.G. Waiting time characteristics in cyclic queues // – Michigan: Probability in the Engineering and Informational Science, – 2004. vol.18, No.03, – p. 299-313.

[20] Синай, Я.Г. Теория фазовых переходов: строгие результаты / Я.Г.Синай – Москва: Наука, – 1980. – 208 с.

can be studied by computer modeling. In this regard, statistical analysis of the numerical outcomes obtained as a result of modeling is required.

Computer modeling, intensive methods of computational statistics, and other modern IT methods of queuing systems are extensive research areas. However, scientists have encountered many severe problems with the simulation of probabilities, such as rare events, traffic jams, various car accidents, and so on.

Sometimes modeling will not allow the collection of observations of rare events. Long-term modeling is required for observation, which is costly, and it is difficult to estimate how long it will take.

As mentioned above, one of the main tools for researching complex systems with moving servers is empirical and computer modeling data and analysis. With computer modeling, we can obtain numerical results of various features of systems. However, statistical analysis of the data is required to make content-rich decisions. Today, special attention is paid to this area. Although this field is essential from scientifically, it is not included in the dissertation topic, and such issues are not considered here. Today, computer methods are widely used to study problems that cannot be solved by analytical methods and ways. The integrated use of analytical and computer methods demonstrates its positive role in scientific research for systems with complex structures and allows them to make effective decisions and make appropriate proposals for practical applications. Such an approach is used in the dissertation, and this approach is used for the analysis of complex structural systems.

Along with computer methods and approaches in this direction, various sections of stochastic processes, such as the results and concepts of point theory of random processes, the results of mass service systems, and others, are widely used. It is worth noting that various random stationary flows, especially Poisson flows, recovery processes, and other concepts, play a unique role in the study of such systems, as experience shows that the demand flow in many practical issues is very close to stationary Poisson flow. Such studies have been conducted[21], and there is still significant interest in this field today.

The collection and analysis of statistical data is an essential problem in the study of complex structured queuing systems. Statistics can indicate the direction in which analytical research is being conducted and may lead to some results. Such studies have been conducted[22].

Among the queuing systems devices of which are in motion, multi-terminal systems and systems with one device that provides sequential service at each terminal are of particular importance. Such systems are widely used in ports and have been researched by various authors.

Objectives and tasks of the study. Queuing systems, devices (servers) of which are in motion, have a random structure, as the demands entering the system, the starting moments of the service, the service life are random processes. Thus, the methods and approaches of theory of probability, stochastic processes, mathematical statistics, and other fields of mathematics stimulate the development of queue theory and allow us to calculate their various features, study different service disciplines and make necessary decisions and recommendations for practical applications.

Many books and articles and even specialized magazines (Queuing systems, Transportation Science, Transportation and Communication, Operation Research, etc.) have been published as this direction has an essential impetus to the development of the world economy. Transportation and Communication, Operation Research, etc.). The widespread use of computers has given a significant impetus to the development of this field, helping to solve problems that are difficult and even impossible to solve analytically.

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[21] Вероятностные и имитационные подходы к оптимизации автодорожного транспорта. / А.П.Буслаев, А.В.Новиков, В.М.Приходько [и др.] – Москва: Мир, – 2003. – 368 с.
Швецов, В.И. Математическое моделирование транспортных потоков // – Москва: Автоматика и телемеханика, – 2003. № 11, – с. 3-46.

[22] Кокс, Д.Р., Смит, В.Л. Теория восстановления / Д.Р.Кокс, В.Л.Смит – Москва: Советское радио, – 1967. – 299 с.

New magazines covering this field have even been published (Journal of Simulation, Simulation Modeling Practice, and Theory, etc.). However, unfortunately, queuing systems with complex structures and devices (servers), particularly the calculation of the optimal movement and parameters of systems, are not widely studied. It is because there is a need to develop new approaches and methods for their study as the structure of mathematical models representing these systems is very complex. Even new directions are emerging in modern times due to the synthesis of other fields of mathematics in this direction. Computer modeling, computer statistics, and other areas can be given as examples. The dissertation pays special attention to these issues.

The main provisions of the defense. The dissertation:

- offers new mathematical models that represent transport systems (automobiles, subways, etc.), communication and communication networks and, in general, queuing systems with moving devices;
- proposes the conditions of various situations arising in the queuing systems with devices in motion;
- proposes a method of selecting the optimal situation according to various parameters;
- proposes a new approach to calculate the various parameters of the proposed models (average waiting time, average service life, etc.);
- gives the calculation of the optimal number of devices that minimize the average standby and service life for different systems.

The scientific novelty of the study. The use of the results of various fields of mathematics (theory of probability and mathematical statistics, stochastic analysis, etc.) is required for the construction of mathematical models of queuing systems with mobile devices and complex structures.

The established mathematical models are new and adequately represent the queuing systems with devices in motion and complex structure and allow us to calculate various parameters. The results obtained can be considered as a continuation and generalization of many results in the literature.

The theoretical and practical significance of the study. The dissertation presents new mathematical models of queuing systems with mobile devices and complex structures. A new approach has been proposed to determine the optimal number and performance of devices in these systems based on the proposed mathematical models. Since these models are universal, they can be used in various fields - transport systems, the optimal structure of communication networks, shipping, and other areas. The theoretical results were published by Springer[23], discussed at international conferences, and won the Grand Prize at The Eighth International Conference on Management Science and Engineering Management in Portugal in 2014.

Approbation and application. The results of the dissertation were approved at the following local and international conferences:

Sixth International Conference on Management Science and Engineering Management (2012), Islamabad, Pakistan;

Actual problems of mathematics and computer science. International conference dedicated to the 90th anniversary of Heydar Aliyev (2013), Azerbaijan, Baku;

Eighth International Conference on Management Science and Engineering Management (2014), Lisbon, Portugal;

Tenth International Conference on Management Science and Engineering Management (2016), Baku, Azerbaijan;

Eleventh International Conference on Management Science and Engineering Management (2017), Kanazawa, Japan;

XXXIII International Conference Problems of Decision Making under Uncertainties (2019), Hurgada, Egypt.

The dissertation was discussed at the extended meeting of the chairs of "Operations Study and Theory of Probability," "Optimization and Management" and "Economic Cybernetics" of Baku State University (2019).

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[23] Abdullayeva, N.A, Hajiyev, A.H., Hasratova, M.H. Mathematical Models of Moving Particals and Their Application for Traffic // Proceedings of the Eighth International Conference on Management Science and Engineering Management (Part of the Advances in Intelligent Systems and Computing book series (AISC, volume 280)), – Lisbon: – 25-27 July, – 2014, – p. 203-214.

The results of the dissertation can be applied in transport systems for optimal traffic.

Name of the organization where the dissertation work is performed. The dissertation work was carried out at the Chair of "Operations Study and Theory of Probability" of Baku State University.

The structure of the dissertation. The dissertation consists of an introduction, three chapters and 76 references. The dissertation in total consists of 107 pages (213605 characters): the First Chapter - 42 pages (83954 characters), the Second Chapter - 18 pages (35967 characters), and the Third Chapter 18 pages (356284 characters).

Abstract. The *introduction* gives a brief summary of study related to the topic of the dissertation in the world scientific literature and notes the important issues in this direction. Besides, the introduction provides a brief summary of the results obtained in the dissertation.

Mathematical models of moving particles (infinite and finite volumes) are constructed and studied in the dissertation. Theory of probability and methods and results of stochastic processes theory are widely used to study them.

The *first chapter* proposes and studies the mathematical models of particles moving on a circle. The models proposed in this chapter are deterministic and show the situations that arise during the motion of particles. Definitions of situations that may arise in different systems are given. The concept of zero state for all systems is explained and the situations that arise in models that start from zero state are defined. It is stated that no circumstances other than those specified here may arise. Necessary and sufficient conditions are found for the occurrence of each of these situations (Theorems 1.2.1; 1.2.2; 1.2.3). A special diagram is proposed to illustrate the situations that may arise in deterministic models (Figure 1.2.3). According to the proposed diagram, the situation after entering the system input parameters is shown. The average waiting time of the entered demands is accepted as the efficiency index reflecting the performance of the systems. This index is defined as the sum of the total waiting time of the entered demands divided by the number of entered demands and allows to compare the situation. The efficiency index is the average waiting time of a particle at a randomly selected point. The average speed of the system, which is closely related to the input efficiency parameter (index), is also determined. A comparison is made between the conditions corresponding to the selected efficiency parameter and the most appropriate (optimal) condition (the condition that receives the minimum value of the efficiency index) is noted. The following theorems are proved in this chapter:

Theorem 1.2.1. For a given $Q_1$and Q_2, to provide the *tsr1* motion mode starting from the *zero position*

$$1 - \min(Q_2 - Q_1, Q_1) < kQ_1 \leq 1$$

or equivalent form

$$1 - \min[(Q_2 - 2Q_1) - \; Q_1] < kQ_1 \leq 1$$

the condition is necessary and sufficient, and the number of such particles k is unique.

Theorem 1.2.2. The following condition is necessary and sufficient for the system (*tsr2*) to start operating from the zero mode:

$$1 < SQ_2 < 1 + (Q_2 - Q_1).$$

All distances between the particles, except for one, must be equal to Q_2, and the distance for only one of them (for example, between the k and the first particle) must meet the following condition:

$$Q_1 < \rho_{k,t} < 2Q_1 < Q_2.$$

Theorem 1.2.3. The following conditions are necessary and sufficient for the system to operate in *Saturated Steady Mode*:

$$kQ_1 < 1 \text{ and } kQ_2 > 1.$$

Theorem 1.3.1. If $\frac{Q_2}{V_2} < \frac{Q_1}{V_1}$, then *(dsr)* motion mode is the optimal mode, i.e., the following is correct:

$$hgm_{dsr} < hgm_{tsr2} \leq hgm_{M(k,k)} < hgm_{tsr1}.$$

Theorem 1.3.2. The following conditions are necessary and sufficient for the system to reach the operating mode (*tsr1*):

$$\frac{1-(Q_2-Q_1)}{Q_1} < k \leq \frac{1}{Q_1}$$

or equivalent form:

$$Q_1 \leq (1-(k-1)Q_1) < Q_2.$$

Theorem 1.3.3. The following conditions are necessary and sufficient for the system to reach the operating mode *(tsr2)*:

$$\frac{1-Q_1}{Q_2} < s < \frac{1+(Q_2-Q_1)}{Q_2}$$

or equivalent form:

$$Q_1 < (1-(s-1)Q_2) < Q_2 + Q_1.$$

Theorem 1.3.4. The following conditions are necessary and sufficient for k_1 and k_2 according to the diagram with the values Q_1 and Q_2 given for the motion of the system (*qsr*) in M (k_1, k_2):

$$k_1 Q_1 + k_2 Q_2 \leq 1,$$

$$k_1 Q_1 + (k_2 + 1) Q_2 \geq 1,$$

$$1 - Q_2 \leq k_1 Q_1 + k_2 Q_2 \leq 1,$$

$$k_2 = max\{s: (s-1)Q_2 + Q_1\} < 1,$$

where $k_1 = k - k_2$,

$$k_{tsr2} < k_{qsr} < k_{dsr} \; and \; \varepsilon_i \to 0, k_{dsr} \to k_{tsr1} \, .$$

The *second chapter* examines a broader class of moving particles and includes the delay function. Introduced delays change the nature of systems into stochastic systems. The function of delays is to forcibly convert the high velocity of a particle to a low velocity. Such delays are called “delay functions” as they delay the waiting time of the moving particles at a selected point on the circle. Input delays include obstacles that may arise in practical matters (for example, road quality in transport systems, unforeseen obstacles on the roads force traffic to slow down, unforeseen conditions in communication systems delay the transmission of information, etc.). Situations arising in stochastic systems are found and compared. It is shown that the results of comparisons are different in stochastic systems (in contrary to deterministic systems). If a predominant (*dsr*) mode of motion is given in deterministic systems, such a situation in stochastic systems is not advisable in practice, as the selected efficiency index of the system receives high values and low average speed values. In stochastic systems, the optimal mode of motion is found according to the intensity of the incoming demand flow (Theorem 2.1.1). It is given that if the optimal mode of motion in deterministic systems is (*dsr*), this mode is inappropriate for stochastic models, and even a stochastic model in such a state quickly switches to the most inappropriate mode (*tsr1*) and never changes from this mode to another. (Theorem 2.1.2).

Theorem 2.1.1. If there is delay in the system, then

a) *(tsr2)* and *(qsr)* modes remain invariant;
b) if

$$Q_1 < \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_k < Q_2$$

then *(dsr)* motion mode changes to *(tsr1)* motion mode and stays there permanently;

c) if

$$\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_k > Q_2$$

then *(dsr)* motion mode changes to *(qsr)* motion mode and stays there permanently.

Theorem 2.1.2. Following relation shall be met (subject to additional conditions):

$$hgm_{tsr2} \leq hgm_{M(k_1,k_2)} \leq hgm_{tsr1} = hgm_{dsr}.$$

Theorem 2.2.1. The following inequalities are true for stochastic systems:

$$hgm_{(tsr2)} < hgm_{(qsr)} < hgm_{(dsr)} < hgm_{(tsr1)}.$$

For $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_k$, which represent the distances between particles in saturated mode is

$$Q_1 < \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_k < Q_2$$

where inequality is true, then the following relation is met:

$$hgm_{(tsr2)} < hgm_{(qsr)} < hgm_{(dsr)} = hgm_{(tsr1)}.$$

Theorem 2.2.2. It is assumed that $t_1, t_2, \dots, t_n$ delays occur at randomly selected times in the system. Besides, it is assumed that, t_1 is a stationary recovery process. If

$$\lambda < \frac{Q_2 - Q_1}{V_2 - V_1}$$

then the only invariant mode among all modes is *(tsr2)*.

The *third chapter* deals with more complex mathematical models of moving particles. It is assumed that each particle has a finite number of volumes, i.e., each particle can serve no more than one demand the number which is not more than *m* at a time. Such models are used in transport (m can be considered as the volume of the vehicle, the memory of devices in communication systems, etc.), communication and computer networks (information that the computer's memory can receive at a time) and other areas. Since such systems are complex, they are studied by computer modeling. The results of modeling are given in the dissertation. Of course, the average waiting time depends on the particle size, depending on the intensity of the demands. As a result of modeling, such a dependence is proposed in tabular form. (Table 3.4.1.). A theorem has been proved in this chapter:

Theorem 3.2.1. If the demand flow included in the system do not depend on the service moments, then the following relations are true for stationary systems:

$$A \to B$$
$$A \leftrightarrow C$$
$$C \to B.$$

The list of literature consists of 76 references and refers to articles and books relevant to the topic of the dissertation in world literature.

CONCLUSION

- In the dissertation, deterministic and stochastic mathematical models of infinitely large particles moving at different speeds on a circle were structured and studied;
- Stationary conditions that can occur in these systems have been found for deterministic models. Finding possible situations allows to calculate the average waiting time of the incoming requirements in such systems and the average speed of the system, which is closely related to it, and to compare different systems according to these indicators;
- A diagram has been constructed for the proposed models, which shows the situation. By entering the system parameters according to the diagram, it is possible to determine the steady state

situations that occur in the system;

- More complex (stochastic) models arise when delays are introduced into the system. The introduced delays are the obstacles in transport, communications and other practical systems. In stochastic systems, ways to calculate the average waiting time of incoming requirements in different motion situations are shown, and numerical examples are given;

- The optimal number of particles for the efficient movement of systems has been determined. It has been shown that if the optimal condition minimizing the waiting time for requirements for deterministic systems is a saturated condition, this is not true for stochastic systems and the optimal condition is (*tsr2*);

- Mathematical models of finitely moving particles are more complex. Results have been obtained for infinite-volume models for such systems.

The main results of dissertation are published in the following academic papers of author:

1. Abdullayeva, N.A. Lift sistemlərinin riyazi modelləri // Azərbaycan Respublikasının Dövlət Müstəqilliyinin Bərpasının 20-ci ildönümünə həsr olunmuş "Riyaziyyatın tətbiqi problemləri" elmi konfransının materialları (XI), – Bakı: – May, – 2011, s. 9-12.
2. Hajiyev, A.H., Abdullayeva, N.A., Mammadov, T.S. Mathematical Models of Queues with Moving Servers: Simple Vertical Transportation Systems // Proseedings of the Sixth International Conference on Management Science and Engineering Management. Part of the Lecture Notes in Electrical Engineering book series (LNEE, volume 185), – Islamabad: – November, – 2012, – p. 529-547.
https://link.springer.com/chapter/10.1007/978-1-4471-4600-1_46
3. Абдуллаева, Н.А.Системы массового обслуживания с рекуррентным обслуживанием // Riyaziyyat və informatikanın aktual problemləri. Heydər Əliyevin anadan

olmasının 90 illik yubileyinə həsr olunmuş beynəlxalq konfransın tezisləri, – Bakı: – May, – 2013, – с. 242-243.
4. Abdullayeva, N.A., Hajiyev, A.H., Hasratova, M.H. Mathematical Models of Moving Particals and Their Application for Traffic // Proceedings of the Eighth International Conference on Management Science and Engineering Management (Part of the Advances in Intelligent Systems and Computing book series (AISC, volume 280)), – Lisbon: – 25-27 July, – 2014, – p. 203-214.
5. Hajiyev, A.H., Mammadov, T.S., Hasratova, M.H., Abdullayeva, N.A. Modeling of Stochastic Vertical Stationary Transportation Systems // Proseedings of the Tenth International Conference on Management Science and Engineering Management. Part of the Advances in Intelligent Systems and Computing book series (AISC, volume 502), – Baku: – September, – 2016, – p. 1271-1278.
https://link.springer.com/chapter/10.1007/978-981-10-1837-4_104
6. Hajiyev, A.H., Abdullayeva, N.A. Optimization of Operating of the Systems with Recurrent Service by Delays // Proseedings of the Eleventh International Conference on Management Science and Engineering Management. Part of the Lecture Notes on Multidisciplinary Industrial Engineering book series (LNMUINEN), – Tokyo: – September, – 2017, – p. 858-867.
https://link.springer.com/chapter/10.1007/978-3-319-59280-0_70
7. Abdullayeva, N.A. Dairə üzərində hərəkət edən hissəciklərin riyazi modelləri // – Bakı: AMEA-nın xəbərləri: fizika-texnika və riyaziyyat elmləri seriyası, – 2018. XXXVIII cild, №6, s. 41-52.
www.icp.az/2018/6-05.pdf
8. Abdullayeva, N.A.Different rejims of motion of moving particles // XXXIII İnternational Conference Problems of Decision Making under Uncertainties (PDMU-2019), – Hurgada: – January, – 2019, – p. 10.

9. Abdullayeva, N.A. On ergodic distribution a cyclical inventory-queuing model with delay // – Bakı: Journal of Contemporary Applied Mathematics, – 2019. vol. 9, No1, – s.3-9. http://journalcam.com/wp-content/uploads/2019/05/1.pdf
10. Abdullayeva, N.A. Stochastic models of moving particles and their application // – Bakı: Məruzələr, – 2019. LXXV cild, №1, – s.29-33.

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